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RANKING OF TOTAL TIME RESERVES FOR DETERMINATION OF THE CRITICAL PATH IN FUZZY NETWORK PLAN

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The Critical Path Method, as a method of network planning, is used in the process of planning and control of complex projects from various fields. For their successful implementation, a clear determination of the duration of each project activity is necessary. However, in construction sector it is very difficult to fulfil this requirement, because the realization of many activities is accompanied by a certain degree of uncertainty and risk. Since construction projects are characterized by uniqueness and unrepeatability, there are often no available historical data for a clear and precise determination of the duration of project activities. Given that, the duration should be predicted by expert and experienced persons, who, in the conditions of unique influencing factors and facing many inaccuracies, should define their decisions during the whole process of project management. Due to the shortcomings of classical planning methods, related to the calculation of the duration of project activities, which does not offer the possibility of modelling complex construction projects, there was a real need for the definition and implementation of new concepts of planning. As an alternative way to model uncertainty and risk, the fuzzy concept was introduced in the planning process, with which the durations of project activities are presented through fuzzy numbers. This fundamental approach, which is based on the application of fuzzy theory, enables advanced and successful modelling of real situations and projects, which overcomes the shortcomings characteristic of classical planning methods. This paper gives an overview of one practical example of application of fuzzy logic in the planning process. One method of ranking fuzzy numbers has been applied for calculation of the earliest and the latest fuzzy time of project activities, and the total time reserves, in order to determine the critical path in fuzzy network diagrams.

Keywords: critical path method, fuzzy logic, risk, uncertainty, construction, planning, time reserve, ranking

1 INTRODUCTION

In modern construction, the realization of any more serious project cannot be imagined without the application of quality planning and management methods. Network models are an example of the most commonly used methods for planning of the implementation of complex and long-term projects. The most commonly used network models in construction are: Critical Path Method (CPM) and the PERT method (Project Evaluation and Review Technique). They are used for analysis of the project time duration, cost and resources. Both methods use a graphical representation of the project through an appropriately oriented network model composed of basic elements such as: activities, events, durations, costs, resources, etc. [4, 10].

The CPM is characterized by a deterministic expression of the duration of the activities in the network plan. This method is applicable for construction projects when the duration of project activities is known, or when it can be precisely determined [4, 10].

The PERT method is used for planning of construction projects when the duration of the project activities is not known or it cannot be explicitly determined. Using the basis of the probability theory the three values for the duration of the project activities can be assess: optimistic, normal and pessimistic duration [4, 10].

When the duration of project activities is deterministic and known, or can be accurately and precisely determined, CPM and PERT methods are useful graphical tools for planning of the project implementation. However, when it comes to planning projects in the field of construction, it is especially difficult, and in some cases even impossible, to accurately predict or determine the duration of certain activities. The occurrence of uncertainty and inaccuracy in the duration of construction activities entails an imprecise determination of the total duration of the project's implementation. In order to overcome these challenges, the application of Fuzzy logic in the process of planning the implementation of projects began, thus Fuzzy network models were developed. Their basic characteristic is that the durations of the project activities are expressed as fuzzy numbers. Through the use of fuzzy theory instead of probability theory, a new, modified critical path method is developed, called the Fuzzy Critical Path Method (FCPM) [4, 7, 9, 10].

2 METHODOLOGY

2.1 Fuzzy critical path method

The application of fuzzy logic in the process of planning and management of construction projects enables the analysis of the duration of construction activities in conditions of uncertainty and risk. The main characteristic of this method is the estimation of the duration of the activities and their expression through triangular or trapezoidal fuzzy numbers. Fuzzy numbers are simply defined as numbers with an uncertain value. Such fuzzy numbers are subject to common algebraic operations according to the laws of algebra, they can be added, subtracted, etc.

Based on the concepts of fuzzy theory, different methods have been developed for fuzzy network planning, which are based on the application of basic fuzzy algebraic operations to calculate the duration of project activities and to determine the total time for the realization of the project and its critical path. Fuzzy network planning gives the possibility to achieve more realistic results and obtain more reliable plan-time and cost project indicators.

The Fuzzy Network Plan, just like the deterministic or classical network plan, can be shown as:

- An activity-oriented network plan (AOA – Activity-On-Arrow network), in which activities are graphically displayed as lines with arrows, and data for duration and resource quantities are entered above or below the arrows, and
- Network plan oriented through events, or Precedence network plan (AON – Activity-On-Node network), in which the activities are graphically displayed as a circle, square, rectangle, etc., in which the data is entered (code, duration, earliest start, earliest end, etc.). Technological dependence between activities is represented by arrow lines.

The procedure for determination of the critical activities and the critical path is explained below, through an example of fuzzy network plan, that is, a network model composed of activities whose duration is expressed using fuzzy numbers. An arrow-shaped network plan view will be used in which activities are represented by arrow lines and are defined by a start and end event (Fig. 1).

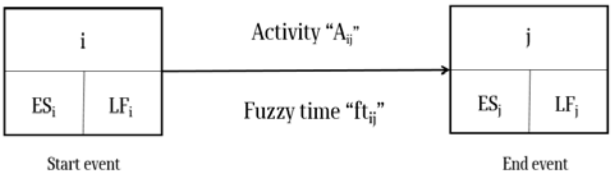

Fig. 1. Graphical representation of an activity in AOA fuzzy network plan [2, 7, 8]

According to the "forward" method, the earliest fuzzy times of completions of the events from all the activities in the network plan are calculated, and the obtained values are entered in the left quarter of the circle with which the event is represented. According to the "backward" method, the latest fuzzy time of completions of the activities from the fuzzy network plan are calculated, i.e. the latest fuzzy execution times of the events from all the activities in the fuzzy network plan, from the last to the first event, and the obtained values are entered in the right quarter of the circle with which event is represented.

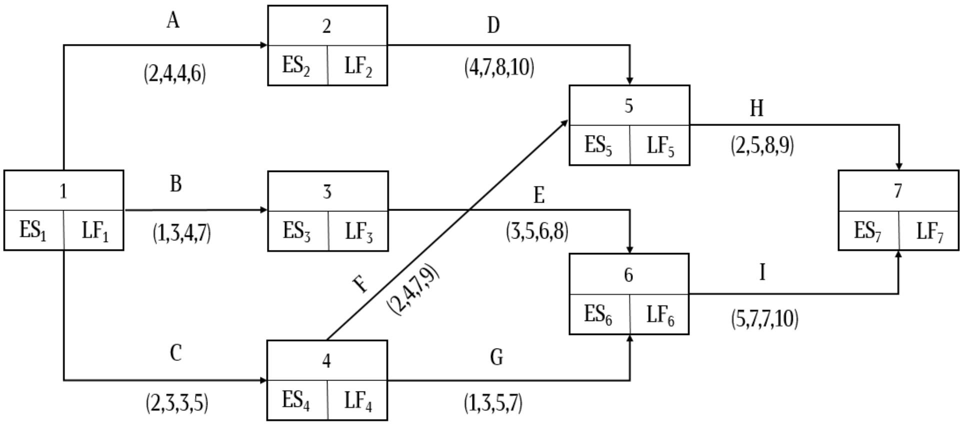

Fig. 2. Graphical representation of a network plan composed of activities whose duration is set with trapezoid fuzzy numbers

In the events in which two or more activities enter or exit, in order to calculate the earliest fuzzy time, or the latest fuzzy time, a ranking, that is, a comparison, of multiple fuzzy numbers must be performed. Ranking of fuzzy numbers is not an easy task at all because the fuzzy numbers are represented by possible distribution functions and can overlap with each other. In the past twenty years a large number of methods for ranking different types of fuzzy numbers have been developed, because from the very beginning of the development and application of fuzzy theory, the analysis and study of the problem related to the comparison of fuzzy numbers began. For comparison, i.e. ranking, of two or more fuzzy numbers, different rules and methods found in professional literature can be used.

In this paper the method of ranking of the total time reserves will be described. The determination of the critical activities and the definition of the critical plan will be explained through the following example of fuzzy network plan (Fig. 2) which is composed of activities whose duration is determined by fuzzy numbers.

2.1.1 Determination of critical path in fuzzy network plan by application of method for ranking of total time reserves

The method of determining the critical path in a fuzzy network diagram by ranking the total time reserves is a method that was first developed by Gin-Shuh Ljang and Tzen-Chen Han. With the help of the formula for ranking of two or more fuzzy numbers it is possible to compare the rank of the fuzzy numbers.

Before starting with the calculation of the rank of the fuzzy numbers, according to the rules of this method, it is necessary to calculate the "*acceptable probability*" with which the planner enters the time analysis. The acceptable probability, marked as β, is calculated as follows [3, 6, 8, 9]:

$$\beta = \left[\sum_i \sum_j \frac{b_{ij}-a_{ij}}{(b_{ij}-a_{ij})+(d_{ij}-c_{ij})}\right]/t \quad (1)$$

where "t" stands for the number of activities included in the network diagram.

For the network plan shown in figure 2, it is obtained that the acceptable risk for the realization of all activities amounts to: β=0.498.

The rank $R(FN_{i,j})$ of the fuzzy number j $FN_{i,j}$ is calculated according to the following expression [3, 6, 8, 9]:

$$R(FN_{i,j}) = \beta\left[\frac{(d_i-x_1)}{(x_2-x_1-c_i+d_i)}\right] + (1-\beta)\left[1-\frac{(x_2-a_i)}{(x_2-x_1+b_i-a_i)}\right] \quad (2)$$

$$\mathrm{x}_1 = \min\,[\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3 \dots\dots \mathrm{a}_\mathrm{n}]$$

$$x_2 = \max\,[d_{1i}, d_2, d_3 \dots\dots d_n]$$

3 RESULTS AND DISCUSSION

3.1 Calculation of the earliest fuzzy time

Event number 1 is the initial event in the network plan, so its earliest fuzzy time is:

$$ES_1 = (0,0,0,0)$$

The earliest fuzzy time for any event "j" in the network plan (except the initial one) is calculated according to the following formula:

$$ES_j = max(ES_i + ft_{i-j})$$

Events 2, 3 and 4 include only one activity each, activities A, B and C respectively, from the event number 1. Their earliest fuzzy times are calculated as follows:

$$ES_2 = ES_1 + ft_{1-2} = (0,0,0,0) + (2,4,4,6) = (2,4,4,6)$$

$$ES_3 = ES_1 + ft_{1-3} = (0,0,0,0) + (1,3,4,7) = (1,3,4,7)$$

$$ES_4 = ES_1 + ft_{1-4} = (0,0,0,0) + (2,3,3,5) = (2,3,3,5)$$

Event number 5 is preceded by two activities: activity D coming from event number 2 and activity F coming from event number 4. Accordingly, the earliest fuzzy time of the occurrence of the event number 5 is calculated as follows:

$$ES_5 = \max\begin{cases}(ES_2 + ft_{2-5}) \\ \\ (ES_4 + ft_{4-5})\end{cases} = \max\begin{cases}(6,11,12,16) \\ \\ (4,7,10,14)\end{cases} = \max\begin{cases}FN_{2,5}, \\ \\ FN_{4,5}\end{cases}$$

The earliest fuzzy time of occurrence of the event number 5 is calculated as the maximum value of the rank of the two fuzzy numbers, which means that to determine which of these two fuzzy numbers is larger, it is necessary to calculate their ranks:

$$\mathrm{x}_1 = \min[6\,,4] = 4;\ x_2 = \max[16,14] = 16$$

$$R(FN_{2,5}) = 0.498\left[\frac{(16-4)}{(16-4-12+16)}\right] + (1-0.498)\left[1-\frac{(16-6)}{(16-4+11-6)}\right] = 0.580$$

$$R(FN_{4,5}) = 0.498\left[\frac{(14-4)}{(14-4-10+14)}\right] + (1-0.498)\left[1-\frac{(16-4)}{(16-4+7-4)}\right] = 0.412$$

Since $R(FN_{2,5}) > R(FN_{4,5})$ it follows that the fuzzy number $FN_{2,5}$ is larger than the fuzzy number $FN_{4,5}$. From here it follows that the earliest fuzzy time of the event 5 is:

$$ES_5 = FN_{2,5} = (6,11,12,16)$$

Using the above-described expressions, the earliest fuzzy times of all events defined in the network plan shown on Figure 2 are calculated. The obtained results are shown in Figure 3.

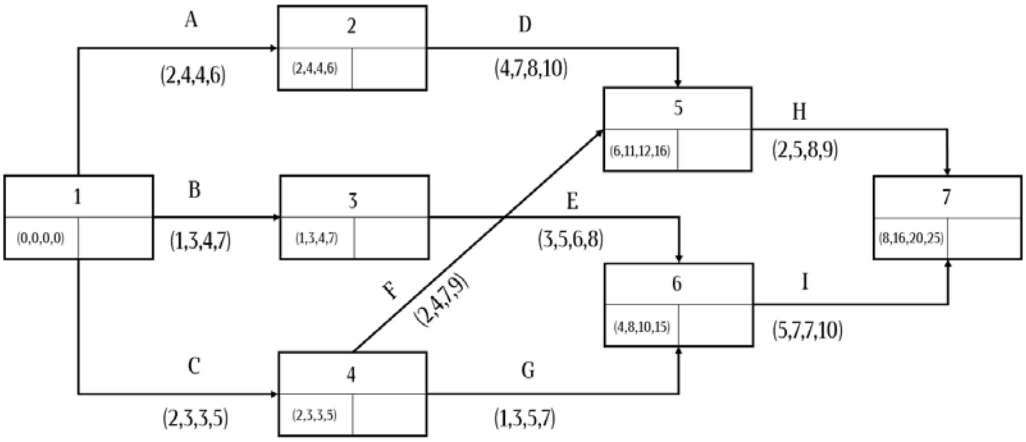

Fig. 3. Earliest fuzzy time of the events in fuzzy network plan

3.2 Calculation of the latest fuzzy time

Event number 7 is the last event in the network plan, so its latest fuzzy time is equal to its earliest fuzzy time:

$$LF_7 = ES_7 = (8,16,20,25)$$

For any other event "j" from the network plan, the latest fuzzy time is calculated according to the following expression:

$$LF_j = min(LF_k - ft_{jk})$$

Event number 7 has two predecessor activities: activity H coming from event number 5 and activity I coming from event number 6. Accordingly, the latest fuzzy times of occurrence of the events number 5 and 6 are calculated as follows:

$$LF_4 = \min\begin{cases} LF_6 - ft_{4-6} \\ \square \\ (LF_5 - ft_{4-5}) \end{cases} = \min\begin{cases} (-2,9,13,20) - (1,3,5,7) \\ \square \\ (-1,8,15,23) - (2,4,7,9) \end{cases} = \min\begin{cases} (-9,4,10,19) \\ \square \\ (-10,1,11,21) \end{cases} = \min\begin{cases} FN_{6,4} \\ \square \\ FN_{5,4} \end{cases}$$

To determine which of these two fuzzy numbers is smaller, the fuzzy number ranking method will be applied:

$$x_1 = min[-9\,,-10] = -10;\ x_2 = max[19{,}21] = 21$$

$$R(FN_{6,4}) = 0.498\left[\frac{(21+10)}{(21+10-11+21)}\right] + 0.502\left[1-\frac{(21+10)}{(21+10+1+10)}\right] = 0.508$$

$$R(FN_{5,4}) = 0.498\left[\frac{(19+10)}{(21+10-10+19)}\right] + 0.502\left[1-\frac{(21+9)}{(21+10+4+9)}\right] = 0.521$$

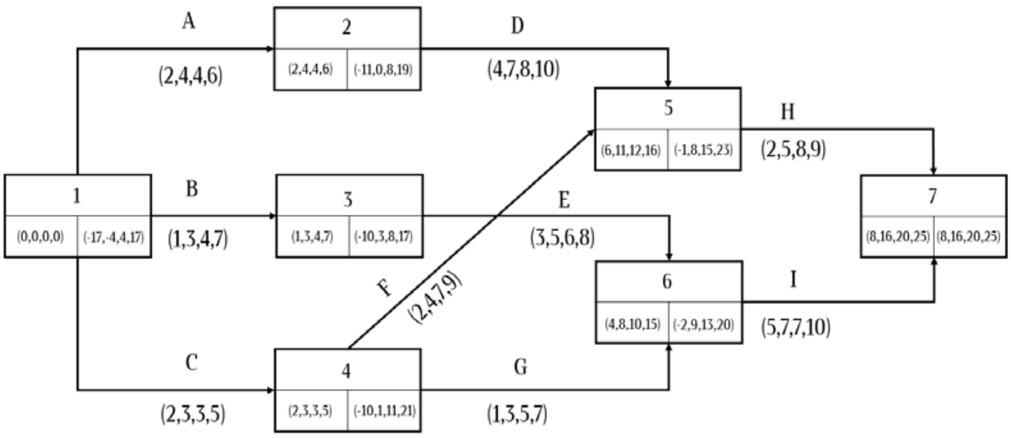

Fig. 4. Latest fuzzy time of the events in fuzzy network plan

Since $R(FN_{6,4}) < R(FN_{5,4})$ it follows that the fuzzy number $FN_{6,4}$ is smaller than the fuzzy number $FN_{5,4}$, thus the latest fuzzy time of occurrence of the event number 4 is:

$$LF_4 = FN_{5,4} = (-10,1,11,21)$$

The latest fuzzy times of all events defined in the network plan are calculated using the same method for raking of fuzzy numbers. The obtained results are shown in Figure 4.

3.3 Determination of the critical path in fuzzy network plan

The interval between the earliest fuzzy execution time of the start event and the latest fuzzy execution time of the end event is the time for which the activity should be executed. If that time has a greater value than the duration of the activity, then that activity has a time reserve, i.e. can be moved within the interval. If that time is the same as the duration of the activity, then that activity is a critical activity, and the value of its time reserve is zero.

The value of the total fuzzy time reserve (fVR_{ij}) for the activity "i-j" is calculated according to the following equation:

$$fVR_{ij} = LF_j - (ES_i + t_{i,j}) \tag{3}$$

According to the rules of the method for ranking of fuzzy number, to determine the critical path in the network plan, it is necessary to calculate the total fuzzy time reserve of all the paths connecting the first and the last event in the network plan. In the given example of a network plan, it can be seen that there are 4 possible paths: $P_1 = (1,2,5,7)$; $P_2 = (1,3,6,7)$; $P_3 = (1,4,5,7)$; $P_4 = (1,4,6,7)$. The calculated values of the total fuzzy time reserves for these 4 possible paths are as follows:

$$VR(P_1) = VR_{1,2} + VR_{2,5} + VR_{5,7} = (-51,-12,12,51)$$

$$VR(P_2) = VR_{1,3} + VR_{3,6} + VR_{6,7} = (-51,-3,15,48)$$

$$VR(P_3) = VR_{1,4} + VR_{4,5} + VR_{5,7} = (-47,-8,20,55)$$

$$VR(P_4) = VR_{1,4} + VR_{4,6} + VR_{6,7} = (-46,-2,20,52)$$

The next step is to determine which of these 4 fuzzy numbers is the smallest, because the path that has the minimum value of the total time reserve is the critical path. The comparison of these fuzzy numbers is done by using the ranking method:

$$\mathrm{x}_1 = \min[-51\,,-51,-47,-46] = -51;\ x_2 = \max[51,48,55,52] = 55$$

$$R(P_1) = 0.498\left[\frac{(51+51)}{(55+51-12+51)}\right] + 0.502\left[1 - \frac{(55+51)}{(55+51-12+51)}\right] = 0.436$$

$$R(P_2) = 0.498\left[\frac{(48+51)}{(55+51-15+48)}\right] + 0.502\left[1 - \frac{(55+51)}{(55+51-3+51)}\right] = 0.511$$

$$R(P_3) = 0.498\left[\frac{(55+51)}{(55+51-20+55)}\right] + 0.502\left[1 - \frac{(55+51)}{(55+51-8+47)}\right] = 0.523$$

$$R(P_4) = 0.498\left[\frac{(52+51)}{(55+51-20+52)}\right] + 0.502\left[1 - \frac{(55+46)}{(55+51-2+46)}\right] = 0.536$$

$$KP = \min[R(P_i)] = min[R(P_1), R(P_2), R(P_3), R(P_4)] = R(P_1)$$

According to the obtained results it can be concluded that the path P_1 (1-2-5-7) is the critical path in the fuzzy network plan, and that the activities 1-2, 2-5 and 5-7 are the critical activities.

4 CONCLUSIONS

In modern construction, it is impossible to imagine the realization of any more serious project without the application of methods for efficient and high-quality planning and control. The main goal of planning is to ensure savings, and to achieve it, it is necessary to precisely determine the duration of the project, which depends on the duration of all activities that are an integral part of the project. Therefore, the basic purpose of the planning process is to ensure the quality assessment of the duration of the project. However, construction projects are often associated with certain risks and uncertainties that can have significant influence on the planned deadlines and project budget and it is important to find a way to include them in the planning process in order to obtain more precise overall results.

In order to include the possible conditions of uncertainty and risk and to model real scenario of construction, the basic concepts of fuzzy theory were implemented in this paper. The durations of project activities are presented through fuzzy numbers and one method of ranking fuzzy numbers has been applied for calculation of the earliest and the latest fuzzy time of project activities, and the total time reserves, in order to determine the critical path in fuzzy network diagrams. The results of the analysis are one positive example that fuzzy theory can be successfully applied in the construction field. The application of fuzzy theory in the process of planning of the realization of construction projects enables advanced and successful modelling of real situations, which overcomes the shortcomings characteristic of classical planning methods.

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