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ESTIMATING DAMAGE PROBABILITY OF THE PRESTRESSED SIMPLE BEAM THROUGH EIGENFREQUENCY MEASUREMENT

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The paper proposes a method for modal-based damage assessment in structures where historical data on their modal properties is often missing. This method is based on the measurements of the structure's eigenfrequencies and predefined assumed damage states. Damage is defined at the level of confirming damage existence with a rough estimate of its extent, within the limits of the predefined assumptions. The factors that affect the outcomes of experimental modal analysis of damaged reinforced and prestressed beam elements on structures in use are briefly outlined and the challenges encountered during these analyses and potential solutions are briefly discussed. The method's application is demonstrated on a damaged prestressed concrete simple beam. Bayesian formulation for probability estimation is used to calculate the probability that a beam, characterized by a specific measured natural frequency, is either undamaged or has a certain level of damage. This calculation is based on the results from finite element models created in the Abaqus software suite. In the finite element models, an eigenfrequency distribution, based on the modulus of elasticity distribution, of a prestressed beam is obtained, for different levels of the assumed prestressing force drop and the corresponding damages caused by the force drop. In the presented problem, the modulus of elasticity of concrete is incorporated into the analysis as an uncertain parameter with a normal distribution. Similarly, other uncertain parameters of the actual structure can be modelled.

Keywords: prestressed beams, frequency-based damage detection, crack distribution, Bayesian probability, modulus of elasticity

1 INTRODUCTION

1.1 Damage detection through modal frequency measurement

Using different damage identification techniques based on changes in dynamic characteristics, through structure vibration measurements and along with modal analysis, the state of the entire structure can be assessed. The application is based on the simple fact that dynamic, modal parameters: modal frequencies, mode shapes and damping, depend on the structure stiffness.

There are a variety of modal-based damage identification methods depending on observed parameters [1–3] and also many probability based [4–6] Certain methods are better suited to certain levels of damage detection - from determining presence only, to determining presence, location, and quantification of damage. Modal, natural or eigenfrequencies are a global characteristic of the structure, and change in their value primarily reveals the existence of damage. Problems arise with the damage spatial localization. For the most accurate detection, a combination of frequency and modal shape analysis is preferred [7–9].

Different methods for damage detection are developed using only frequency change [10–12]. An attractive feature of frequency-only damage detection methods is that frequencies are relatively easy to measure. The limitations of this method are reflected in the fact that significant damage can cause a small change in frequencies, especially if measurements are made on larger structures. Damages in the areas of relatively high stresses result in a significant reduction of eigenfrequencies, while in the case the damage is located in the area of low stresses, its detection this way can be unreliable [13]. Measurement equipment characteristics and signal processing methods can also cause unprecise results [14]. Moreover, often, the reduction or rise in eigenfrequencies might result from environmental and operational conditions [15]. Given the significant impact on the outcomes, it's advisable to conduct long-term observations and track alterations in modal characteristics, which would help in identifying the effects of these conditions, with the ultimate goal of mitigating their influence. Considering all the above, a minimum change of 5% in natural frequency is roughly suggested in practice, to reliably identify damage existence [9].

In the case of prestressed beams, for detecting the drop in prestressing force, the problem is even more complex. In a case of a force drop, the degree and type of prestress affect changes in eigenfrequencies in different ways, depending on the characteristics of the specific system [15–17]. If non-bonded steel for prestressing is used, in the ideal case, the "softening effect" applies, and an increase in frequencies comes with the decrease in force, but, if the steel is sealed with injection mass, this effect does not apply, and in the general case, a drop in frequencies with a drop in force is observed [18], [19]. Also, the impact of prestressing forces on the modal properties is often relatively minor compared to the impact exerted by other parameters.

The objective of the analysis presented in this paper is to assess the extent of damage on a specific structural element, namely a prestressed concrete beam, by considering various hypothetical damage states and using the measured frequency value of the 1st vertical mode of oscillation.

2 METODOLOGY

2.1 Prestressed beam damage probability estimation

Hereafter, a calculation to estimate the probability that a specific beam has a presumed damage state is presented. Based on the analysis of a beam with the predicted damage, and known frequency value - the result obtained from the measurements, Bayesian formulation was used to determine the posterior probability of damage.

For rough estimation, five damage states were modeled, which gave predicted behavior. The damage states were determined by assuming a uniform decrease in the prestressing force, ranging from full prestressing force on an undamaged beam (100%P), to a complete loss of prestressing force (0%P). The damage is reflected in the drop of the prestressing force, and the appearance of corresponding cracks.

In all variants, adopted reinforcement and geometric and material characteristics are the same. The modulus of elasticity of concrete was considered to have a normal distribution, which, in this case, was the only uncertain parameter under consideration, out of many that can occur in practice.

Finally, based on the adopted distribution of the modulus of elasticity of concrete, the frequency distribution for each beam with different damage state was obtained by numerical calculation. Frequency distributions were used to calculate probability estimation.

3 RESULTS AND DISCUSSION

3.1 Analyzed prestressed beam characteristics

3.1.1 Beam parameters used in analysis

A concrete element with a length of 10 m, a cross-section of 65/30 cm, hinged support (simple beam) was analyzed. The steel bar for prestressing Y1100H was used, the required parameters of which are defined by the standard *EN 10138-4* [20], and the concrete used is class 40/50. The undamaged, prestressed beam is dimensioned according to *EN 1992-1-1* [21]. Based on the initial prestressing force P of 130 kN, and initial prestressing losses of 20%, 9 profiles of prestressing rods in the lower zone, arranged in 2 rows, were adopted.

The dispersion of the *characteristic* strength of the existing concrete depends on quality control of the embedded concrete and on the concrete class, based on which certain values of dispersion parameters are given [22]. The material strength dispersion parameters are correlated with the elastic modulus, which is an input parameter in finite element models.

According to [22], it can be assumed that the relations between the estimated *in situ* and the *characteristic* compressive strength, as well as the corresponding coefficients of variation, are:

$$E[f_c]=0.675\cdot f_{ck}+7.7\leq 1.15\cdot f_{ck}, \tag{1}$$

$$COV_{fc}^2=COV_{fck}^2+0.0084, \tag{2}$$

where f_c is *in situ*, a f_{ck} the *characteristic* compressive strength, COV_{fc} and COV_{fck} corresponding coefficients of variation, and $E[f_c]$ the expected strength value.

Expected value for tangent modulus of elasticity of concrete, $E[E_{ct}]$, for a certain class of concrete, can be expressed as a function of its in situ compressive strength [22]:

$$E[Ect]=5.015\sqrt{E[f_c]} \tag{3}$$

For *characteristic* compressive strength $f_{c,k}$ = 40 MPa, the expected *in situ* strength value, according to expression (1) is $E[f_c]$= 34.7 MPa, and for average quality control ($COV_{fc,k}$ =0.15, [22]), the coefficient of variation, according to the expression (2) is COV_{fc} = 9.52 %. Based on (3), the mean value of the tangent modulus of elasticity is $E[E_{ct}]$ = 29 542 MPa, and the corresponding coefficient of variation is COV_{Ect} = 8.1 %.

As strength and modulus of elasticity are directly proportional quantities with a normal distribution, the cumulative distributions of these variables have the same shape (Fig. 1).

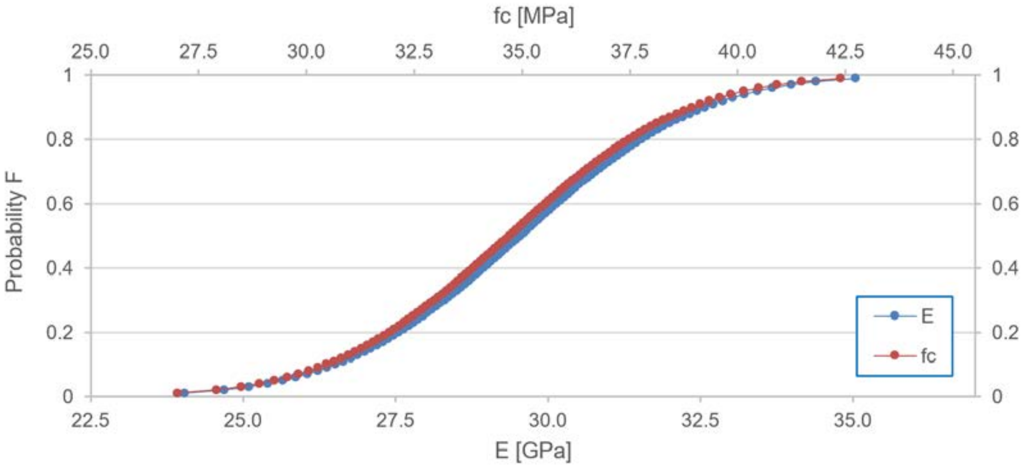

Fig. 1. Cumulative distribution of strength and modulus of elasticity of concrete C 40/50

3.1.2 Adopted characteristics of damage

Five damage states were considered on five beam models, with corresponding prestress force 100%P, 75%P, 50%P, 25%P, 0%P, labeled as B_1 to B_5, respectively. The steel bars of the modeled beam in the example are connected to the concrete, and the cracks are modeled as empty volumes in the concrete. At different prestressing forces, a simplified calculation of the cracks was performed, and their distribution and width were roughly given, based on the algorithm defined by the standard EN 1992-1-1 [14]. The neutral axis, as well as the distribution and width of the cracks, were calculated for the values of the corresponding moments, for each beam individually. It was assumed that the cracks extend to the position of the neutral axis. Also, it was adopted that the cracks from the maximum moment in the beam spread in the middle quarter of the span L, and then they decrease gradually, according to the corresponding moments at lengths of L/8. The zones of crack decrease are shown in Fig. 2.

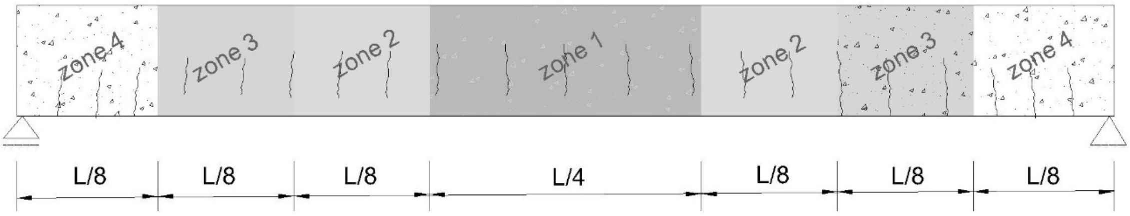

Fig. 2. Arrangement of different zones for determining crack dimensions

With every reduction in the prestressing force, there is a noticeable increase in both the total area of cracking and the width of the cracks, as well as a decrease in the spacing between the cracks. Crack widths range from 0.386 to 0.317 mm, in all 4 zones, for B_5, and up to 0.129 mm, only in zone 1 for B_2. The crack distribution in the numerical models are shown in Fig. 3.

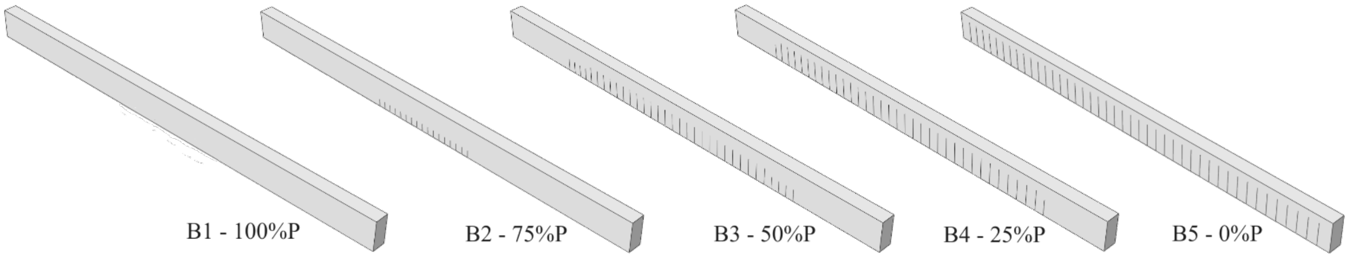

Fig. 3. Crack representation for a different level of prestressing force drop

3.2 Finite element model parameters

Abaqus FEA software package was used for frequency calculation in models representing damaged beams. A 25 mm finite element mesh was adopted. The prestress in the models was introduced by defining the coefficient of thermal expansion of steel, and by the temperature change value, *dT*. The change in temperature *dT* is determined by following the value of the calculated stresses. The deformation after the applied prestress, as well as the 1st oscillation mode of the beam with a 50% prestressing force drop, is shown in Fig. 4.

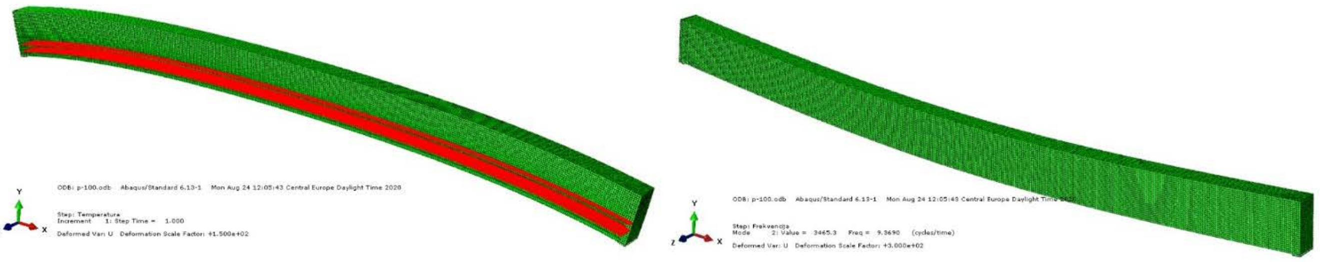

Fig. 4. Display of deformations caused by prestressing force (left) and the 1st vertical mode of oscillation (right) for a beam with 50% prestressing force drop

3.3 Frequency distribution of first vertical mode

Following the previously defined normal distribution of the modulus of elasticity, determined for a beam with average quality control of embedded concrete, the program calculates the corresponding frequencies of the 1st vertical mode of oscillation for different damage states (B_2 to B_5) and for an undamaged state (B_1). So, for each model individually, corresponding eigenfrequency values were obtained based on the distributed values of the modulus of elasticity.

Based on modulus of elasticity distribution, normal distribution is also assumed for eigenfrequency values, and control was performed for all beam models, using the method of probability paper for normal distribution. In Fig. 5, the control for B_1 and B_5 is presented, where F^{-1} is inverse cumulative distribution function, and f is frequency in Hz. Straight line of the plot confirms the assumption.

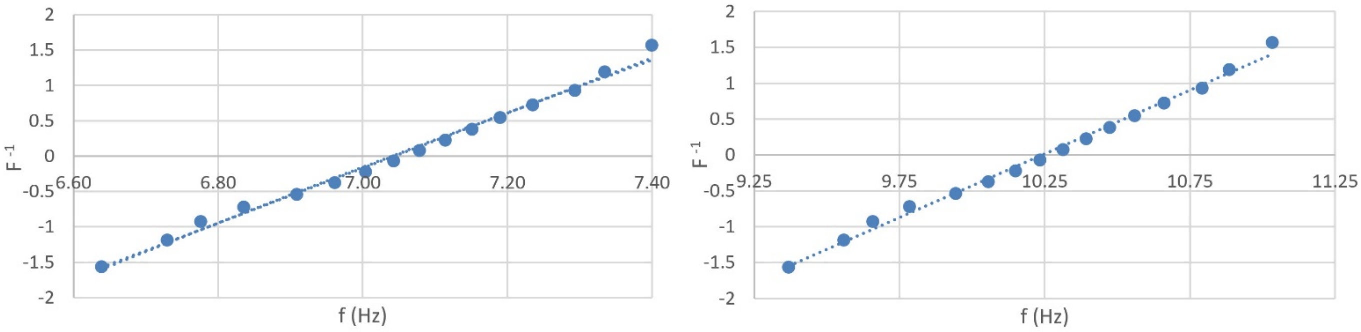

Fig. 5. Control of the adopted normal distribution of the obtained frequencies by probability paper method for the normal distribution for B_1 (100%P) (left) and B_5 (0%P) (right)

The mean values $\overline{X}$ [Hz], standard deviations S [Hz], coefficients of variation COV [%] and frequency ranges in Hz, for the obtained distributions of five models are given in Table 1.

Table 1. Parameters of normal frequency distributions

	Beam 1	Beam 2	Beam 3	Beam 4	Beam 5
$\overline{\mathbf{X}}$ [Hz]	10.24	8.99	7.24	7.09	7.04
S [Hz]	0.475	0.384	0.232	0.216	0.217
COV [%]	4.64	4.28	3.20	3.05	3.08
f range [Hz]	1.67	1.35	0.81	0.76	0.76

The probability distribution densities of frequencies, are shown in Fig. 6.

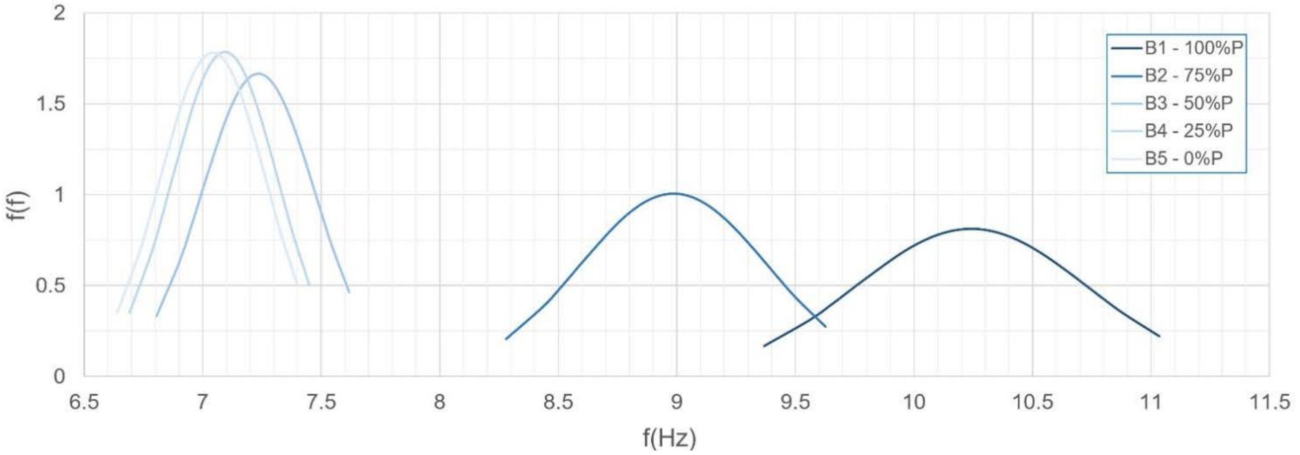

Fig. 6. Probability distribution density of frequencies f(f) of modelled beams

It was observed that, with the increase in the degree of damage, the mean value of the frequencies of the 1st mode of oscilation decreases. The adopted uniform drop of prestressing force did not gave uniform propagation of cracks, nor a proportional drop in mean frequency values. It is obvious that the frequency of the 1st vertical mode is greatly influenced by both parameters that were taken into consideration, the modulus of elasticity of concrete, and the dimensions and arrangement of cracks. For the same distribution of elastic modulus, beams with less damage show a higher frequency variation. Beams with a loss of 75% prestressing force - B_4 and a complete loss of prestressing force - B_5, show very similar values of normal distribution parameters.

3.4 The probability of damage state determination with measured frequency F

The probability that a beam with a certain degree of damage (B_i) has a frequency f, is denoted by $\mathrm{P(f|Bi)}$, where i is from 1 to 5. In another words, this notation denotes the probability of an event f (in this case, a certain frequency) occurring, given that another event B_i (a beam with a certain degree of damage) has already occurred, meaning, the probability of encountering the frequency f when dealing with the beam Bi.

If a specific observation or measurement result is 9.5 Hz, there is no possibility that the drop in the prestressing force is greater than 25%, as can be seen in Fig. 6. Therefore, based on the obtained distributions for B_1 and B_2, it is possible to calculate probabilities $\mathrm{P(f|B_1)}$ and $\mathrm{P(f|B_2)}$, by calculating the areas under the corresponding distribution density diagrams in the region of the measured frequency (9.5 Hz), as shown in Fig. 7.

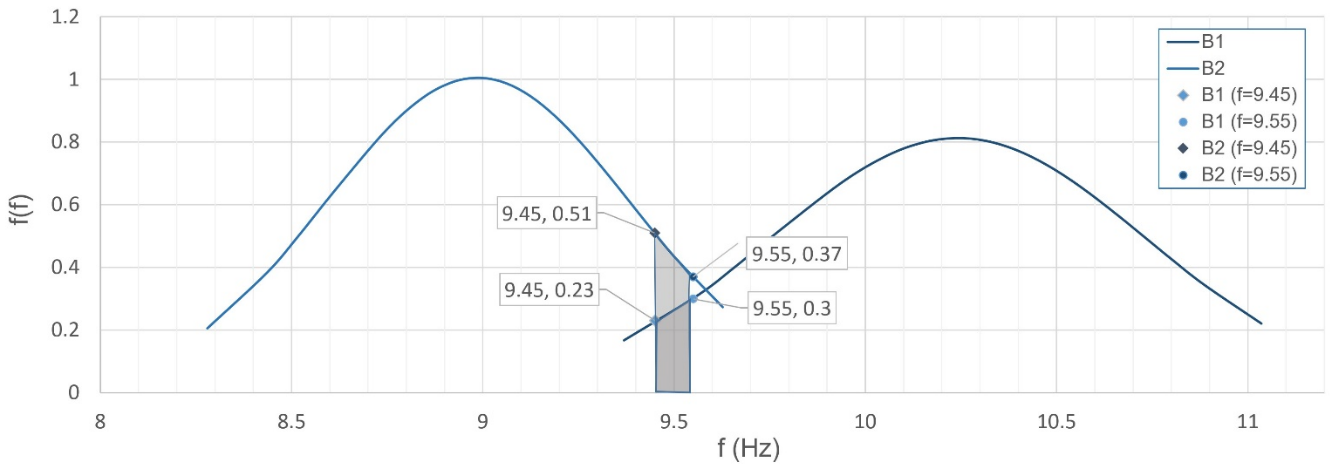

Fig. 7. Determination of probability $P(f|Bi)$

Probabilities that this frequency measurement corresponds to B_1 or B_2, denoted as $P(9{,}5|B_1)$ and $P(9{,}5|B_2)$, are found to be 2.65% and 4.40% respectivly.

If the measured value was 8 Hz, it would suggest that the extent of damage is between B_2 with 25%, and B_3, with 50% reduction in prestressing force.

Finally, the probability that the beam is in a particular state of damage - B_i, given that a frequency f has been measured, is equal to:

$$\mathrm{P(B_i|f)} = \frac{\mathrm{P(f|B_i)\cdot P(B_i)}}{\mathrm{P(f)}} = \frac{\mathrm{P(f|B_i)\cdot P(B_i)}}{\mathrm{P(f|B_1)\cdot P(B_1)+P(f|B_2)\cdot P(B_2)+P(f|B_3)\cdot P(B_3)+P(f|B_4)\cdot P(B_4)+P(f|B_5)\cdot P(B_5)}}, \quad (4)$$

where is:

$\mathrm{P(B_i|f)}$ – (posterior) probability that it refers to beam *i*, if the measured frequency is *f*,
$\mathrm{P(B_i)}$ – (prior) probability that it refers to beam *i*,
$\mathrm{P(f)}$ – the probability that one of the considered beams has a measured frequency *f*,
$\mathrm{P(f|B_i)}$ – the probability that a certain beam *i* has a measured frequency *f*.

Table 2 shows the prior probabilities that a certain beam is one of 5 beams with different damage states, $\mathrm{P(B_i)}$, which are considered to be equal, as well as probabilities that a beam with a certain degree of damage has a measured frequency of f = 9.5 Hz - $\mathrm{P(9.5|B_i)}$, which are calculated above.

Table 2. Damage characteristics of beams with different degrees of prestressing force

	Beam 1	Beam 2	Beam 3	Beam 4	Beam 5
$\mathbf{P(B_i)}$	0.2	0.2	0.2	0.2	0.2
$\mathbf{P(9.5|B_i)}$	0.0265	0.0440	0	0	0

Using the expression (4) and values from Table 2, the probability that it refers to B_1 (undamaged beam), if the measured frequency is 9.5 Hz, $\mathrm{P(B_1|9.5)}$= 0.376, is obtained, while the probability that it is a beam B_2 (beam with a 25% drop in prestress force) is $\mathrm{P(B_2|9.5)}$= 0.624.

The specific frequency measurement suggests a decrease in the beam's prestressing force, indicating a higher probability of a 25% reduction rather than the beam being undamaged. Based on the results, a smaller force drop than 25% can be assumed, and we can proceed to further analysis.

4 CONCLUSIONS

With the aim of more reliable damage detection on existing structures, through the eigenfrequency measurements, it is desirable to have as much previous data as possible, obtained by continuous or periodic target measurements. As such data, often, does not exist, this paper presents a way of assessing a damage existence using current eigenfrequency measurements, simulating structural element with several predicted damage states, and applying Bayesian method for posterior probability.

Using material property distribution, frequency distribution corresponding to a 1st oscillation mode is obtained, for five modelled damage states of the prestressed simple beam. Based on these distributions, and using the Bayesian formulation with certain simplifications, a calculus is given for determining the probability that a beam with a specific measured value of eigenfrequency, has a defined damage. Thus, considering calculated probabilities, the extent of damage was concluded, but only roughly, within the limits of the modelled damage states.

Every numerical model representing a real structure carries with it a degree of unreliability, in terms of unknown material properties or structural element’s parameters. In the case of prestressed beams, the modelling problem is quite complex, as the degree and type of prestressing affect changes in eigenfrequencies in different ways, depending on the characteristics of the specific system. But, since the actual extent of damage is unknown, it's crucial to incorporate in the model as many known properties as possible. To obtain such data, it is preferable to carry out field tests, but, even so, many properties remain uncertain. In the example given herein, with no testing data, the modulus of elasticity of concrete was entered into the analysis as an uncertain parameter with a normal distribution, and his significant impact on the results is demonstrated, especially in the case of the undamaged beam, and beam with 25% loss in the prestressing force. A similar way, other unreliable parameters can be introduced, such as the stiffness of the supports, or the position of the prestressing route.

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